How is transversity related to helicity
for quarks and antiquarks inside the proton?

Claude Bourrely

Département de Physique, Faculté des Sciences de Luminy,
Université de la Méditerranée - Aix-Marseille II,
13288 Marseille, Cedex 09, France

Franco Buccella

Dipartimento di Scienze Fisiche, Università di Napoli,
Via Cintia, I-80126, Napoli and INFN, Sezione di Napoli, Italy

Jacques Soffer

Physics Department, Temple University
Barton Hall, 1900 N, 13th Street
Philadelphia, PA 19122-6082, USA

Abstract

We consider the quark and antiquark transversity distributions inside a polarized proton and we study how they are expected to be related to the corresponding helicity distributions, both in sign and magnitude. Our considerations lead to simple predictions in good agreement with their first determination for light quarks from experimental data. We also give our predictions for the light antiquarks transversity distributions, so far unknown.

Our understanding of the proton spin structure has greatly improved over the last twenty years or so due, on the one hand, to significant progress on the theoretical side and on the other hand, to several dedicated experiments at CERN, DESY, JLab and SLAC on polarized deep inelastic scattering (DIS). More recently, the advent of the polarized pp collider at RHIC-BNL has opened up a new era for a better knowledge of the proton spin structure and also for testing the spin sector of perturbative QCD.

The main source of information on the internal proton structure lies in the parton distributions. If $\mathcal{A}(x)$ denotes the quark distribution in a proton, as a density matrix in both the quark and proton spin, it will be expressed in terms of direct products of two Pauli matrices σ_i and the unit matrix I. Then, by choosing the z-axis along the proton momentum, the x-axis and y-axis normal to it, $\mathcal{A}(x)$ reads

$$\mathcal{A}(x) = q(x)I \otimes I - \Delta q(x)\sigma_z \otimes \sigma_z - \delta q(x)(\sigma_x \otimes \sigma_y + \sigma_y \otimes \sigma_x) , \qquad (1)$$

where $q(x)$ is the *unpolarized* distribution, whereas for the *polarized* distributions, one must distinguish helicity distributions inside a longitudinally polarized proton, denoted for quarks by $\Delta q(x)$ and transversity distributions inside a transversely polarized proton, denoted by $\delta q(x)$. These last two distributions are required for a complete description of the quark spin in the proton at leading twist.

The vast programme of unpolarized DIS data taking at HERA has led to a rather precise determination of the quark (q), antiquark $(\bar{q})$ and gluon (G) unpolarized distributions, which is very relevant to study hadronic processes at LHC-CERN [1]. The helicity distributions have been determined so far, with a reasonable precision level, because they can be directly extracted from polarized DIS but this is not the case for the transversity distributions, which are not easily accessible, being chiral odd, they decouple from DIS [2]. However, there is a strong bound on transversity, resulting from positivity and derived a few years ago [3]; it involves helicity and reads

$$q(x) + \Delta q(x) \geq 2|\delta q(x)| , \qquad (2)$$

for quarks and similarly for antiquarks, which is obviously more severe for negative quark helicity distributions.

Let us first examine some arguments to appreciate the relevance of this positivity constraint [1]. In the non-relativistic limit, transversity and helicity

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[1]For a recent review on positivity constraints for spin observables see Ref. [4].

coincide indeed, that is

$$\delta q(x) = \Delta q(x) \ , \tag{3}$$

so the bound is trivially fullfilled, provided $\Delta q(x) \geq 0$. On the contrary if $\Delta q(x) \leq 0$, the bound implies

$$\Delta q(x)/q(x) \geq -1/3 \ . \tag{4}$$

This simple remark is stressing the importance of the sign of $\Delta q(x)$ and we now turn to review what is known about this sign, for the different flavor quarks and antiquarks. Concerning the light quarks helicity distibutions, it is well established that, $\Delta u(x) > 0$ and $\Delta d(x) < 0$, and according to some very accurate JLab data [5], the ratio $\Delta d(x)/d(x)$ is close to -1/3 at $x = 0.6$, with a possible violation of Eq. (3) [2], which leads to a contradiction with the assumption Eq. (3). This trend has been correctly predicted by our quantum statistical approach for unpolarized and polarized parton distributions [7], where the non-diffractive part, the only one contributing to helicity distributions, is given by Fermi-Dirac functions, namely the first term in the r.h.s of Eq. (14) of [7] for quarks and the first term of the r.h.s of Eq. (15) for antiquarks. The parton distributions determined in [7] have been successfully compared with new experimental results in [8, 9]. Concerning the light antiquark helicity distributions, the statistical approach imposes a strong relationship to the corresponding quark helicity distributions. In particular, it predicts $\Delta \bar{u}(x) > 0$ and $\Delta \bar{d}(x) < 0$, with almost the same magnitude, in contrast with the simplifying assumption $\Delta \bar{u}(x) = \Delta \bar{d}(x)$, often adopted in the literature. The COMPASS experiment at CERN has measured the valence quark helicity distributions, defined as $\Delta q_v(x) = \Delta q(x) - \Delta \bar{q}(x)$. These recent results displayed in Fig. 1 are compared to our prediction and the data give $\Delta \bar{u}(x) + \Delta \bar{d}(x) \simeq 0$, which implies either small or opposite values for $\Delta \bar{u}(x)$ and $\Delta \bar{d}(x)$. Indeed $\Delta \bar{u}(x) > 0$ and $\Delta \bar{d}(x) < 0$ are predicted both by the chiral quark soliton model (CQSM) [11]-[13] and the statistical approach [7] and it leads to a positive contribution of the sea to the Bjorken sum rule [14].

Although strange quarks and antiquarks s and $\bar{s}$ play a fundamental role in the nucleon structure, they are much less known than the parton distributions for the light quarks u and d. For completeness, let us just mention that we have extended the statistical approach to this case and we have found that

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[2]It is interesting to note that in Ref. [6], one argues that $0 \geq \Delta d(x)/d(x) \geq -1/3$ for all x, based on general properties of a three quark bound state obeying the Pauli principle.

$\Delta s(x)$ and $\Delta \bar{s}(x)$ are both negative for all x values [16]. Negative values are also predicted by the flavor SU(3) CQSM [15], with the magnitude of $\Delta \bar{s}(x)$ much smaller than that of $\Delta s(x)$. The conclusion on recent COMPASS data [17] is that $\Delta s(x)$ is nearly zero or slightly negative depending on the choice of the kaon fragmentation functions.

So to summarize this discussion about the sign of the quark and antiquark helicity distributions, it seems clear that $\Delta u(x) > 0$ and $\Delta d(x) < 0$, whereas for the remaining ones the sign is not yet firmly established which requires a more elaborate future experimental investigation.

Let us now recall that, by studying the effects due to the Melosh-Wigner rotation [18, 19], an *approximate* relation between transversity and helicity distributions was derived, namely

$$\Delta q_{RF}(x) + \Delta q(x) = 2\delta q(x) \ , \tag{5}$$

where $\Delta q_{RF}(x)$ measures the quark constituent spin in the proton rest frame, which differs from $\Delta q(x)$ due to the Melosh-Wigner rotation. It looks similar to Eq. (2) and it is compatible with it, when $\Delta q(x) \geq 0$, since $q(x) \geq \Delta q_{RF}(x)$. One can make another interesting remark to stress the difference between u-quarks and d-quarks. By considering the first moments, according to SU(6), one has

$$\Delta u_{RF} = 4/3 \qquad \text{and} \qquad \Delta d_{RF} = -1/3 \ , \tag{6}$$

whereas the first moments at $Q^2 = 0$ are

$$\Delta u = 2F \qquad \text{and} \qquad \Delta d = F - D \ , \tag{7}$$

where F and D are the two parameters introduced by Cabibbo in his famous work on the weak current of the hadrons [20], whose values are $F = 0.464$ and $D = 0.787$. Therefore

$$\Delta u < \Delta u_{RF} \qquad \text{and} \qquad \Delta d \simeq \Delta d_{RF} \ . \tag{8}$$

This is reasonable, since the Melosh-Wigner rotation depends on the transverse momentum of the quarks, which is expected, in the framework of the quantum statistical approach, to be larger for the u-quarks than for the d-quarks. This is due to the fact that the u-quark distribution, which has the largest first moment, occupies broader regions of the phase-space both in the longitudinal and transverse directions [21].

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So by taking $\Delta d(x) \simeq \Delta d_{RF}(x)$ and by using Eq. (5), for the d-quark, which gives $\Delta d(x) \simeq \delta d(x)$, we see that the positivity bound Eq. (2) implies again $\Delta d(x)/d(x) \geq -1/3$. So we have some evidence that the assumption Eq. (3) combined with the Jlab data [5] might lead to a violation of the positivity bound for the d-quark and we propose to assume instead

$$\delta q(x) = \kappa \Delta q(x) \ , \tag{9}$$

where κ is a normalization factor which will be taken at the largest value, in such a way that the positivity bound is satisfied for the d-quark. By using the statistical distributions [9], we show in Fig. 2 the resulting quark transversity distributions for $Q^2 = 2.4\text{GeV}^2$, where we took $\kappa = 0.6$, for u and d flavors, together with the positivity bounds. Although this value of κ was chosen to satisfy positivity for the d-quark, for simplicity, the same value was kept for the u-quark. In Ref. [22], the first extraction of the u- and d-quark transversity distributions had been obtained, by a combined global analysis of the azimuthal asymmetries in semi-inclusive polarized DIS measured by HERMES at DESY and COMPASS at CERN and those in $e^+e^- \rightarrow h_1 h_2 X$ unpolarized processes by the Belle Collaboration at KEK. The agreement between these experimental results [22] and the curves displayed in Fig. 2 is rather satisfactory, within the uncertainties. One should note that since $\kappa < 1$, at least at low Q^2, transversity is *smaller* than helicity, in agreement with Ref. [22], but in contrast with the results of Ref. [19], which was predicting transversity *larger* than helicity. Moreover, in Ref. [19] the signs of the quark transversity distributions is correctly predicted, but the magnitudes are far too large. A comparative analysis of transversity and helicity was also presented in a more recent work [23], with some predictions from the CQSM, and although the d-quark transversity distribution has the correct sign and magnitude, it is definitely too large for the corresponding u-quark.

Another relevant point, not yet mentioned so far, concerns the Q^2 evolution because it is important to take into account the different Q^2 evolution of the transversity and helicity distributions as discussed recently in Ref. [24]. In particular, one should remember that the scale dependence of the tensor charges is fairly strong in sharp contrast to the case of the axial charges. This means that the simplifying assumption Eq. (9) at the initial scale, becomes an approximation at higher scales, an obvious limitation for very accurate predictions, because κ should depend on Q^2.

For completeness, by using Eq. (9) for antiquarks, we also give the resulting transversity distributions displayed on Fig. 3, which have the same signs as

the quark transversity distributions and satisfy positivity. As direct consequence of this, the double transverse spin asymmetry A_{TT}, for Drell-Yan muon pair production is expected to be positive, for pp collisions as well as for $\bar{p}p$ collisions. We show in Fig. 4 the expected asymmetries at RHIC-BNL for pp and at the new FAIR accelerator complex at Darmstadt for $\bar{p}p$, where there are speculations for producing a polarized $\bar{p}$ beam [25]. In both cases the asymmetry increases with larger M. Clearly we expect the corresponding double helicity asymmetry A_{LL} to be larger as already pointed out in several other cases [26], a theoretical observation which must be carefully checked by experiments.

Finally, it is interesting to remark that, at least at low Q^2, the quark transversity and helicity distributions have rather similar shapes, which was not necessarily anticipated. Our knowledge on the transversity distributions is at the earlier stage and we look forward to precise data, to improve this important aspect of the proton spin structure.

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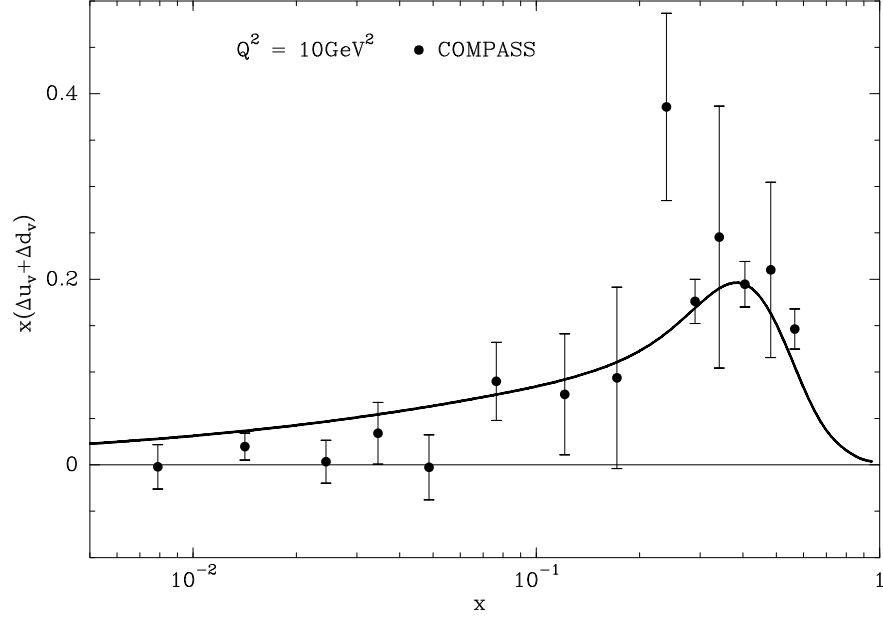
Figure 1: The valence quark helicity distributions, versus x and evolved at $Q^2 = 10\mathrm{GeV}^2$. The solid curve is the prediction of the statistical approach and the data points come from Ref. [10].
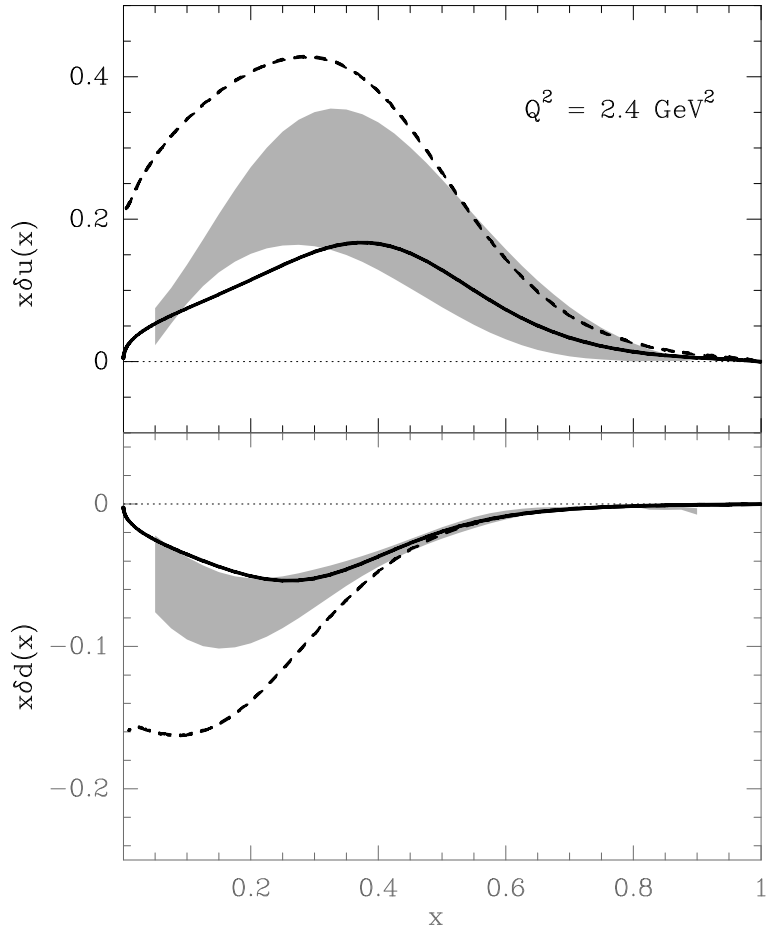
Figure 2: The resulting quark transversity distributions for u and d flavors, as a function of x for $Q^2 = 2.4\text{GeV}^2$. The dashed lines are the positivity bounds and the shaded areas are the uncertainties bands obtained in Ref. [22].
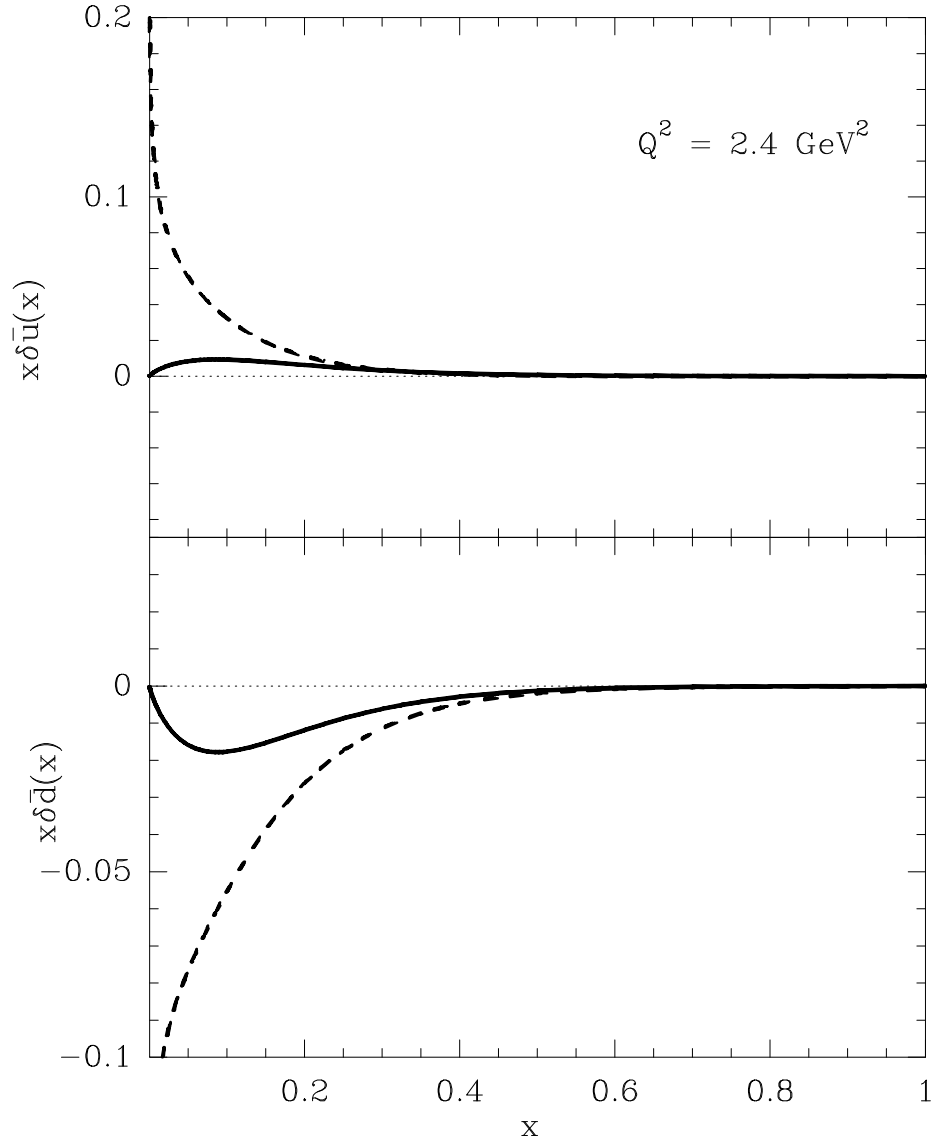
Figure 3: The resulting antiquark transversity distributions for u and d flavors, as a function of x for $Q^2 = 2.4\mathrm{GeV}^2$. The dashed lines are the positivity bounds.
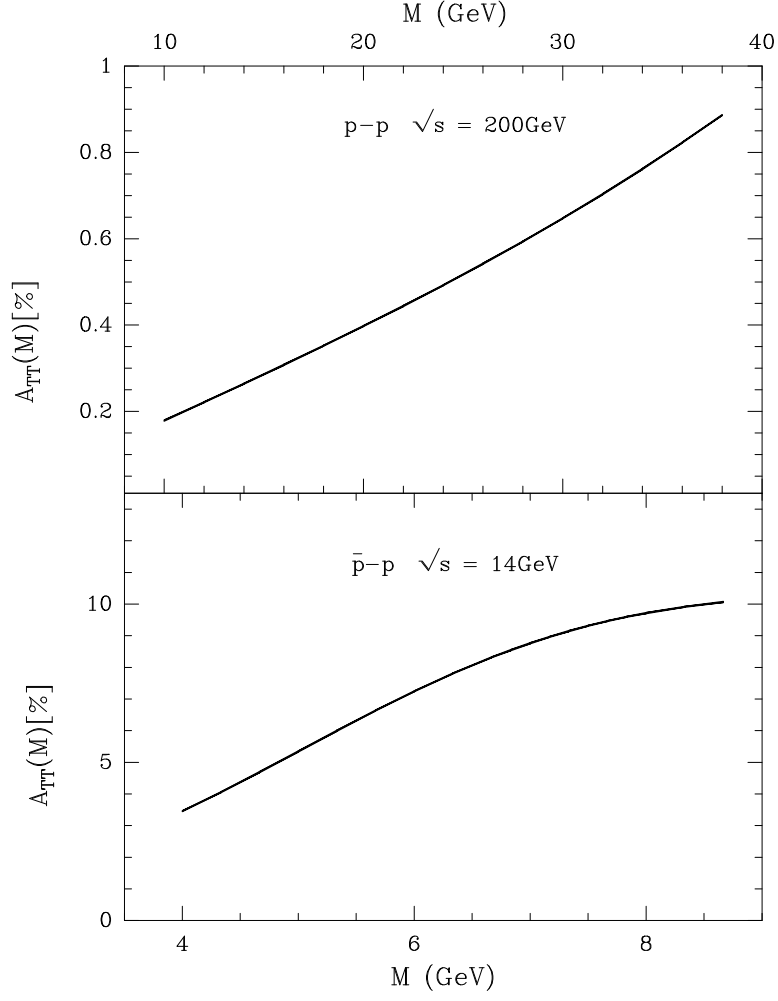
Figure 4: The predicted double transverse spin asymmetry for Drell-Yan muon pair production at zero rapidity, versus the muon pair mass M. Upper part for pp collisions at RHIC-BNL. Lower part for $\bar{p}p$ collisions at FAIR Darmstadt.